Modeling of Return Strokes with Their Initiation Processes under Consideration

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We developed an engineering return stroke model specifically for studying the effects of return stroke initiation processes. In the model, two current waves with identical wave shapes propagate upward and downward at two independent speeds from a height-variable return stroke initiation point. Using that model and several results recently observed on lightning return stroke initiation processes, we have studied the electromagnetic fields produced by a return stroke at a horizontal distance ranging from several tens of meters to several tens of kilometers from the strike termination point. It is shown that, depending on the front steepness of the current waveform, the presence of the downward return stroke current wave may result in a visible initial peak in the rising portion of the return-stroke-produced field waveforms. A larger initiation height appears to result in an initial field peak with a larger amplitude, and also longer duration time. Moreover, a larger downward return-stroke speed tends to produce a sharper initial field peak.

***Index Terms*—Electromagnetic field, lightning, return stroke, return-stroke modeling, transmission line (TL) model.**

I. Introduction

RECENTLY it was reported that a return stroke with a larger peak current usually initiates higher above the ground [1-3]. For a natural first return stroke with a peak current of 76.3 kA, an initiation height of about 90 m was reported [3]. Once a return stroke initiated from a height, according to Wang *et al.* [1–3], it propagates upward and downward usually at two different speeds. Sometimes, the downward speed could be as small as 2.2×10^7 m/s, about 5 times smaller than a typical return stroke speed (upward). Apparently, all these factors which form the so-called return stroke initiation processes could influence the relation between the return stroke electric current and the return stroke electric or magnetic field.

In the present paper, we proposed an extended return-stroke model which can take account of all the recent observation results made by Wang *et al.* [1–3], and thus can reproduce field waveforms in better agreement with the measured ones than those previous models.

II. Theory and Model

For our extended return-stroke model that takes account of the initiation process, a vertical channel over a perfectly conducting ground is assumed. The upward return stroke and downward return stroke waves, with identical current waveforms, propagate from the initiation height, h_{ri}, to the ground and to the cloud, with the constant speed v_u and v_d, respectively, as shown in Fig. 1.

Using the temporal and spatial current distribution along the return stroke channel, $I(z', t)$, specified by the return-stroke model, the vertical electric field E_z and horizontal magnetic field B_φ, at ground level and at a horizontal distance r from the channel are calculated by

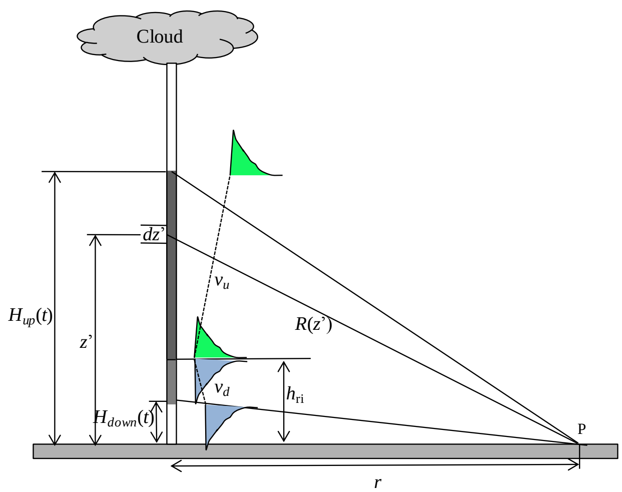

Fig. 1. Geometry used in deriving the expressions of apparent height of return-stroke front and apparent return-stroke speed at a point P on ground for the initiation process with the initiation height of h_{ri}. Two return strokes propagate upward and downward from the junction point with the constant speed v_u and v_d, respectively.

$$E_z(r,t)=\frac{1}{2\pi\varepsilon_0}\int_{H_B(t)}^{H_T(t)}\frac{2z'^2-r^2}{R^5(z')}\int_{\frac{z'}{v_f}+\frac{R(z')}{c}}^{t}I(z',\tau-R(z')/c)\,d\tau\,dz' + \frac{1}{2\pi\varepsilon_0}\int_{H_B(t)}^{H_T(t)}\frac{2z'^2-r^2}{cR^4(z')}I(z',t-R(z')/c)\,dz' - \frac{1}{2\pi\varepsilon_0}\int_{H_B(t)}^{H_T(t)}\frac{r^2}{c^2R^3(z')}\frac{\partial I(z',t-R(z')/c)}{\partial t}dz' \qquad (1)$$

$$B_\varphi(r,t)=\frac{\mu_0}{2\pi}\int_{H_B(t)}^{H_T(t)}\frac{r}{R^3(z')}I(z',t-R(z')/c)\,dz' + \frac{\mu_0}{2\pi}\int_{H_B(t)}^{H_T(t)}\frac{r}{cR^2(z')}\frac{\partial I(z',t-R(z')/c)}{\partial t}dz' \qquad (2)$$

where ε_0 is the permittivity, μ_0 is the permeability of free space, c is the speed of light. $H_B(t)$ and $H_T(t)$ are the bottom and top of the corresponding channel, and obtained as

$$H_{up}(t) = v_u \cdot [h_{ri} / v_u + t - (r^2/c^2 + h_{ri}{}^2/c^2 + t^2 v_u{}^2/c^2 + 2h_{ri} v_u t/c^2 - r^2 v_u{}^2/c^4)^{1/2}] / [1-(v_u/c)^2]. \quad (3)$$

$$H_{down}(t) = v_d \cdot [h_{ri} / v_d - t + (r^2/c^2 + h_{ri}{}^2/c^2 + t^2 v_d{}^2/c^2 - 2h_{ri} v_d t/c^2 - r^2 v_d{}^2/c^4)^{1/2}] / [1-(v_d/c)^2]. \quad (4)$$

We considered in this study two channel-base current waveforms corresponding, respectively, to the first and subsequent return strokes, using the multiple Heidler functions. The transmission line (TL) model, in which the current waves propagate along the channel without any distortion or attenuation, is adopted here not only because the TL model is one of the most simplest models, but also it has been reported to function reasonably well in reproducing both close and relatively distant fields for the initial few microseconds of return strokes in rocket-triggered lightning.

III. Analysis and Results

Vertical electric fields at a distance of $r = 5$ km are plotted in Fig. 2 for full scale value of 10 μs. The contributions of the electrostatic, induction and radiation components of the upward and downward return strokes are also included. A sharp initial peak, which do not appear on the corresponding current waveform, is superimposed on the rising portion of the electric and magnetic fields of the subsequent stroke. The magnetic fields, for both subsequent and first strokes, although not presented here due to the limited paper space, are dominated by the induction fields coming from the closest current and, hence, the waveforms are similar to the channel-base current. An initial peak is superimposed on the magnetic field of the subsequent stroke, but not on the first stroke.

Using the extended model, we have compared predicted electromagnetic fields at a horizontal distance ranging from several tens of meters to several tens of kilometers from the lightning channel. Besides the front steepness of the current waveform, the initiation height and the downward return-stroke speed are also influencing factors of characteristics features of such a peak. Our results show that a larger initiation height appears to result in an initial field peak with a larger amplitude, and also longer duration time. A larger downward return-stroke speed leads to a sharper initial field peak.

Interestingly, the sharp initial peak has been found, for return strokes of triggered lightning, in the measured 5.15-km and 20-km electrical field by Willett *et al*. [4], as well as in 50-m electrical field by Leteinturier *et al*. [5], none of the above corresponding measured channel-base current exhibiting similar initial peaks. In the final presentation, more detailed results will be reported.

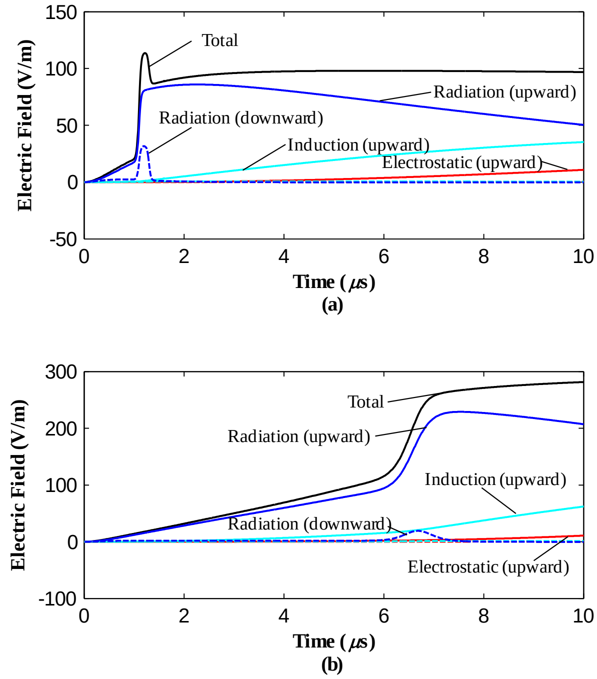

Fig. 2. Calculated vertical electric fields for (a) subsequent return stroke, (b) first return stroke at a distance r = 5 km within 10 μs. The upward and downward return-stroke speed are assumed to be 2×10^8 m/s and 1×10^8 m/s, respectively. The junction point is at height h_{ri} = 20 m.

IV. Conclusions

Depending on the front steepness of the current waveform, the presence of the downward return stroke may result in a visible superimposed sharp initial peak in the rising portion of the field waveforms, which cannot be observed from the corresponding current waveforms.

These superimposed initial peaks, which cannot be produced by the conventional models that the current wave is assumed to be launched from the ground, may be found in the measured fields by cloud-to-ground lightning.

In the case of the channel-base current with a small front steepness (such as a maximum rate of rise of 25 kA/μs in this study), no distinguishable superimposed initial peak can be found in the predicted field wavefroms by the extended model.

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